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MATHEMATICS
HIGHER LEVEL
PAPER 3 – CALCULUS

Thursday 15 May 2014 (afternoon)

1 hour

INSTRUCTIONS TO CANDIDATES

- Do not open this examination paper until instructed to do so.
- Answer all the questions.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A graphic display calculator is required for this paper.
- A clean copy of the ***Mathematics HL and Further Mathematics HL*** *formula booklet* is required for this paper.
- The maximum mark for this examination paper is *[60 marks]*.

Please start each question on a new page. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. *[Maximum mark: 16]*

Consider the functions f and g given by $f(x) = \frac{e^x + e^{-x}}{2}$ and $g(x) = \frac{e^x - e^{-x}}{2}$.

(a) Show that $f'(x) = g(x)$ and $g'(x) = f(x)$. *[2]*

(b) Find the first three non-zero terms in the Maclaurin expansion of $f(x)$. *[5]*

(c) Hence find the value of $\lim_{x \to 0} \frac{1 - f(x)}{x^2}$. *[3]*

(d) Find the value of the improper integral $\int_0^{\infty} \frac{g(x)}{[f(x)]^2} dx$. *[6]*

2. *[Maximum mark: 17]*

(a) Consider the functions $f(x) = (\ln x)^2$, $x > 1$ and $g(x) = \ln(f(x))$, $x > 1$.

(i) Find $f'(x)$.

(ii) Find $g'(x)$.

(iii) Hence, show that $g(x)$ is increasing on $]1, \infty[$. *[5]*

(b) Consider the differential equation

$$(\ln x)\frac{dy}{dx} + \frac{2}{x}y = \frac{2x - 1}{(\ln x)}, \; x > 1.$$

(i) Find the general solution of the differential equation in the form $y = h(x)$.

(ii) Show that the particular solution passing through the point with coordinates (e, e^2) is given by $y = \frac{x^2 - x + e}{(\ln x)^2}$.

(iii) Sketch the graph of your solution for $x > 1$, clearly indicating any asymptotes and any maximum or minimum points. *[12]*

3. *[Maximum mark: 12]*

Each term of the power series $\frac{1}{1\times 2}+\frac{1}{4\times 5}x+\frac{1}{7\times 8}x^2+\frac{1}{10\times 11}x^3+\ldots$ has the form $\frac{1}{b(n)\times c(n)}x^n$, where $b(n)$ and $c(n)$ are linear functions of n.

(a) Find the functions $b(n)$ and $c(n)$. *[2]*

(b) Find the radius of convergence. *[4]*

(c) Find the interval of convergence. *[6]*

4. *[Maximum mark: 15]*

The function f is defined by $f(x)=\begin{cases} e^{-x^2}\left(-x^3+2x^2+x\right), & x\le 1 \\ ax+b, & x>1 \end{cases}$, where a and b are constants.

(a) Find the exact values of a and b if f is continuous and differentiable at $x=1$. *[8]*

(b) (i) Use Rolle's theorem, applied to f, to prove that $2x^4-4x^3-5x^2+4x+1=0$ has a root in the interval $]-1, 1[$.

(ii) Hence prove that $2x^4-4x^3-5x^2+4x+1=0$ has at least two roots in the interval $]-1, 1[$. *[7]*